A NEW APPROACH TO OBTAIN RAMBERG-OSGOOD CONSTITUTIVE EQUATION FOR MATERIALS USED IN STRUCTURAL COMPONENTS OF NUCLEAR REACTORS

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Abstract

The paper develops a new approach to obtain the strain-hardening exponent for the flow curves in tensile of the structural materials that are further used in the constitutive equation describing stress-strain behaviour. The stress-strain experimental curve obtained at room temperature by testing in the air the stainless steel 316 L was fitted as the Ramberg - Osgood constitutive equation. The paper describes the experimental test methodology and the theoretical approach concerning the strain-hardening exponent, as well as the indications to solve numerically the transcendental respective equation. A comparison between the curves: the experimental stress-strain, predicted by modelling and those obtained with FEACrack software is discussed. The work belongs to the integration effort of the methodology developed in the framework of the R&D CANDU Fuel Channel Program, to obtain the material properties for the Generation IV innovative materials, most of them planned to be used in the ALFRED components.

Keywords: *Structural materials, Strain-hardening, Constitutive equation, Material properties*

Introduction

In the research area for the Generation IV nuclear reactors [1], a major task is to manage the structural integrity of the nuclear components under representative environment conditions. Thus, the target to confirm a demanding safety of functioning and operation for the nuclear facilities by considering the aggressive and damaging factors as irradiation, high temperature, and corrosion, constitutes a priority even from the design stage.

To simulate the mechanical behaviour of the structural materials from various components of the nuclear reactor in Generation IV, finite element analyses are widely used. For this, there is a need for the description of the strain-hardening behaviour of materials using mathematical expression between mechanical stress and strain of materials. Also, in application related to the fracture mechanics parameters, the stress-strain equations are required for the evaluation of crack tip conditions. Thus, in the finite element analyses for the extinction of finite elements from the front of the crack to simulate crack initiation and propagation, a fracture mechanics parameter, *J*-integral will be used, and this needs flaw curves of the material as input.

In the present paper, the stress-strain constitutive equation (Ramberg- Osgood type) is obtained based on the results from the experimental curve from the testing of 316L stainless-steel in air. The work is part of the efforts to integrate the methodology developed within the R&D CANDU Fuel Channel Program coordinated by RATEN ICN, to obtain the material properties for Generation IV innovative materials, most of them planned to be used in ALFRED components. In this respect, the experimental tests of specific ″dog-bone″ specimens, prepared from the CANDU pressure tube, Zr-2.5%Nb alloy, were performed in previous works.

Next, a new approach to obtain the strain-hardening exponent for the flow curve in the tensile of the metallic materials is proposed. Later on, this approach is applied to the stress-strain experimental curve obtained at room temperature by testing in air 316L specific tensile specimen. The strain-hardening exponent is used in the Ramberg - Osgood constitutive equation. The paper describes the experimental test methodology and the theoretical approach concerning the strain-hardening exponent, as well as the indications to solve numerically the nonlinear respective equation. A comparison between the experimental stress-strain curve, predicted by modelling, and also those obtained with FEACrack software is discussed.

Ramberg – Osgood constitutive equation

The engineering stress-strain curve does not give a true indication of deformation characteristics of metal because it is based on the original dimensions of the specimens, and these dimensions change continuously during the test [2].The metal continues to strain-harden all the way up to fracture, therefore that the stress required to produce further deformation should also increase. In case that true stress, based on the actual cross-sectional area of the specimen is utilized, and so the strain is also based on the instantaneous dimensions, the curve which is obtained is known as a true-stress versus true-strain curve. This is often also known as flow curve since it represents the basic plastic-flow characteristics of the material.

The true stress (σ) and true strain (ε) is defined as [2]:

$$\sigma = \frac{P}{A_0}(1+e) \tag{1}$$

$$\varepsilon = \ln(1+e) \tag{2}$$

$$e = \frac{\Delta L}{L_0} \tag{3}$$

with: P- instantaneous load; A_0 - original cross-sectional area of the specimen; e - engineering strain; ΔL - total specimen deformation; L_0 - initial specimen length.

The mathematical relationship which expresses the true stress-strain curve is a power curve [2, 3]. The Ramberg - Osgood flow rule states that, for quasi-static loading at a given temperature and strain rate, the true stress σ is given as a function of the strength coefficient K, strain-hardening exponent n, and true plastic strain ε_P as:

$$\sigma = K\varepsilon_p^n \tag{4}$$

where: n - is the strain-hardening coefficient; K - is the strength coefficient.

The instability in tension loading occurs into a specimen is known as the necking phenomenon. Necking generally begins at maximum load during the tensile deformation of a ductile metal [2]. Thus, the necking or localized deformation begins at maximum load, where the increase in stress due to a decrease in the cross-sectional area of a specimen becomes greater than the increase in the load-carrying ability of the metal due to strain hardening [2]. Let us consider the definition of stress:

$$P = \sigma \cdot A \tag{5}$$

where: P - mechanical load; σ - stress; A - cross - sectional area of the specimen.

The condition of instability resulting in localized deformation is defined by condition:

$$dP = \sigma \cdot dA + A \cdot d\sigma = 0 \tag{6}$$

The constancy of specimen volume: $\frac{dL}{L} = -\frac{dA}{A}$ (7)

gives: $-\frac{dA}{A} = \frac{dL}{L} = d\varepsilon_P$, ε - the strain (8)

From Eqs. (6) and (7) results:

$$\frac{d\sigma}{\sigma} = d\varepsilon_P \ \ or, \frac{d\sigma}{d\varepsilon_P} = \sigma \tag{9}$$

From Eq. (4) it is obtained: $\frac{d\sigma}{d\varepsilon_P} = nK\varepsilon_P^{n-1}$ (10)

By combining Eq. (9) with (10), we get: $\sigma = nK\varepsilon^{n-1}$

It is possible to determine the strain at which the necking occurs: $\sigma = K\varepsilon_P^n = nK\varepsilon_P^{n-1}$ (11)

This means that the strain at which necking occurs ε_u is numerically equal to the strain hardening coefficient [2]:

$$\varepsilon_u = n \tag{12}$$

Based on this assertion, the present paper proposes a new method to obtain the strain hardening coefficient, from the stress-strain curve. The relationship (4), which describes the flow curve, could be written for yield point pair (σ_y, ε_y) and the necking pair (σ_u, ε_u):

$$\sigma_y = K \cdot \varepsilon_y^n \tag{13}$$

Respectively, $\sigma_u = K \cdot \varepsilon_u^n$ (14)

From relationships (13) and (14) is obtained:

$$\frac{\sigma_u}{\sigma_y} = \left(\frac{\varepsilon_u}{\varepsilon_y}\right)^n \tag{15}$$

and, by using logarithm function:

$$\ln\left(\frac{\sigma_u}{\sigma_y}\right) = n\left[\ln(\varepsilon_u) - \ln(\varepsilon_y)\right] \tag{16}$$

Considering relationship (12), results in a transcendental equation for strain hardening coefficient:

$$\ln\left(\frac{\sigma_u}{\sigma_y}\right) = n\left[\ln(n) - \ln(\varepsilon_y)\right] \tag{17}$$

Eq. (17) is numerically solved in the mathematical computing software Matlab[4] for the present paper.

The Ramberg - Osgood Eq. (4) could even be put in another form, which is mostly utilized within the finite element software. Starting from Eq. (12), results:

$$\varepsilon_p = \left(\frac{\sigma}{K}\right)^{\frac{1}{n}} \tag{18}$$

and this becomes: $\varepsilon - \varepsilon_e = \left(\frac{\sigma}{K}\right)^{\frac{1}{n}}$ (19)

where: ε - the total strain; ε_e - the elastic strain.

But the elastic strain is: $\varepsilon_e = \frac{\sigma}{E}$ (20)

with E - Young modulus of elasticity. Using (20) in (19):

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{\frac{1}{n}} \tag{21}$$

By introducing the next new parameters

$$\left(\frac{1}{K}\right)^{\frac{1}{n}} = \alpha \cdot \frac{\sigma_0^{1-\frac{1}{n}}}{E} \tag{22}$$

The Ramberg - Osgood equation becomes:

$$\varepsilon = \frac{\sigma}{E} + \alpha \cdot \frac{\sigma_0}{E} \cdot \left(\frac{\sigma}{\sigma_0}\right)^{\frac{1}{n}} \tag{23}$$

with: α - material parameter; n - strain-hardening coefficient; σ_0 - reference stress.

Finally, the last format of Ramberg - Osgood equation is

$$\frac{\varepsilon}{\frac{\sigma_0}{E}} = \frac{\sigma}{\sigma_0} + \alpha \cdot \left(\frac{\sigma}{\sigma_0}\right)^{\frac{1}{n}} \tag{24}$$

or

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \cdot \left(\frac{\sigma}{\sigma_0}\right)^{\frac{1}{n}} \tag{25}$$

Eq. Ramberg - Osgood (25) is completely determined if α and n are known. By solving Eq. (17), the strain hardening coefficient n is obtained. To obtain α, in Eq. (25) we take $\sigma = \sigma_0 = \sigma_y$, and results:

$$\varepsilon - \frac{\sigma_y}{E} = \alpha \cdot \frac{\sigma_y}{E} \tag{26}$$

As σ_y is the yield stress with the plastic deformation of 0.2%, results:

$$\alpha = \frac{0.002 \cdot E}{\sigma_y} \tag{27}$$

From the experimental stress-strain curve we determine α and n parameters, and the equation Ramberg - Osgood as well in the format from (25).

The experimental tests in air environments

The 316L standard-size specimen was tested in air at the strain rate of $10^{-3}s^{-1}$ and room temperature according to ASTM E8/8M [5].

The Walter Bai Testing Facility is displayed in Fig. 1(a) and, the typical specimen aspect after testing can be seen in Fig. 1(b).



Fig. 1. a) Walter Bai Testing Facility for tensile test; b) 316L specimen after testing in air

After the experimental tensile test in air, the methodology proposed in the previous chapter to obtain the strain-hardening coefficient n and the material parameter α from the experimental stress-strain curve. Further, the constitutive equations, in the Ramberg-Osgood format are obtained.

In the air, the Ramberg - Osgood constitutive equation for 316L at room temperature is:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + 2.18 \cdot \left(\frac{\sigma}{\sigma_0}\right)^{5.78} \tag{28}$$

In Fig. 2, a comparison between the experimental stress-strain curve and those from the Eq. (28) is displayed for the air environment.

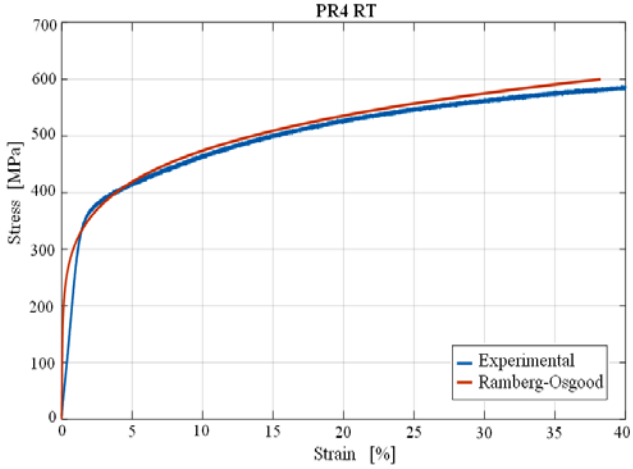


Fig. 2. Comparison between experimental and Ramberg Osgood stress-strain relationships for the tensile test in the air at room temperature

By using the algorithm which is built-in the FEACrack software, the Ramberg - Osgood constitutive equation for 316L at room temperature is:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + 2.18 \cdot \left(\frac{\sigma}{\sigma_0}\right)^{4.70} \quad (29)$$

In Fig. 3 a comparison between the experimental stress-strain curve and those from the Eq. (29) is displayed.

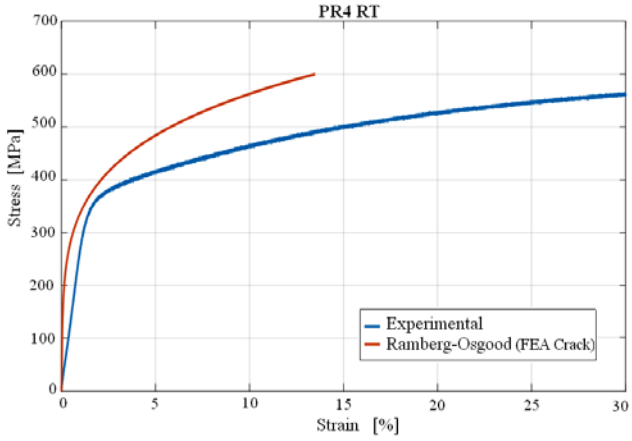


Fig. 3. Comparison between experimental and Ramberg Osgood from FEACrack for the tensile test at room temperature

It can be seen from the comparisons between the experimental curves from Fig. 2 and Fig. 3, the following assertions:

- The Ramberg - Osgood constitutive equation (Eq. 28) obtained using a new approach proposed in the paper give a quite good prediction of the stress-strain behaviour in tensile for the 316L specimen tested in the air;
- The Ramberg - Osgood constitutive equation (Eq. 29) obtained using the algorithm, which is built-in the FEACrack software, give a weaker prediction of the stress-strain behaviour in tensile for the 316L specimen tested in the air.

Few comments are necessary at this point. These results could be considered in the range of inherent variability parameters resulted from the experiments.

Therefore, future activities are focused to perform a real database containing many similar tests, with very good confidence in the material properties, specimen dimension accuracy measurements, and stability of the testing parameters.

These will lead to the possibility of statistical use of data, and further to obtain a generalization of Ramberg - Osgood equation, with confidence intervals for practical use.

Conclusions

A new approach to obtain the strain-hardening exponent for the flow curve in tensile of the structural materials that are further used in the constitutive equation describing stress-strain behaviour is proposed.

The stress-strain experimental curve obtained at room temperature by testing in the air the stainless steel 316 L was converted as the Ramberg - Osgood constitutive equation.

The paper describes the experimental test methodology and the theoretical approach concerning the strain-hardening exponent, as well as the indications to solve numerically the transcendental respective equation.

A comparison between the experimental stress-strain curve, predicted by modelling and that obtained with FEACrack shows that the proposed method gives a good agreement. This confirms our possibility of using the database, which is in progress at RATEN ICN, and further to obtain a generalization of Ramberg - Osgood equation, with confidence intervals for practical use.

The work belongs to the integration effort of the methodology developed in the framework of R&D CANDU Fuel Channel Program, to obtain the material properties for the Generation IV innovative materials, most of them planned to be used in the ALFRED components.

The model is useful in the structural integrity activities of the innovative materials that will be used in the ALFRED demonstrator, planned to be built in Romania, on RATEN ICN nuclear platform.

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